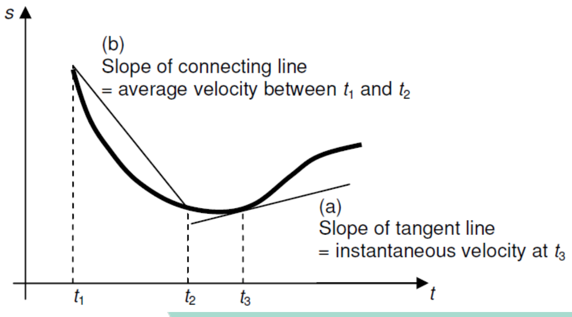
Chapter 2: Kinematics

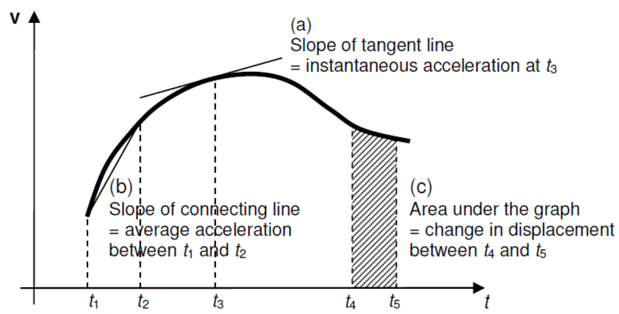
1. Speed, Displacement, Velocity, Acceleration

The **displacement** of a particle is its change of position.
The **velocity** of a particle is the rate of change of its displacement.
The **acceleration** of a particle is the rate of change of its velocity.
The **average speed** of a particle is defined as the distance moved divided by the time taken.
The **average velocity** is defined as the displacement moved divided by the time taken.
Note that average speed is a scalar quantity, while displacement and average velocity are vectors.

Graphically,

- Velocity is the gradient of the displacement-time graph.
- Acceleration is the gradient of the velocity-time graph.
- Displacement is the area under the velocity-time graph.
- Velocity is the area under the acceleration-time graph.





In particular, for a motion with constant acceleration a and initial velocity u, we have:

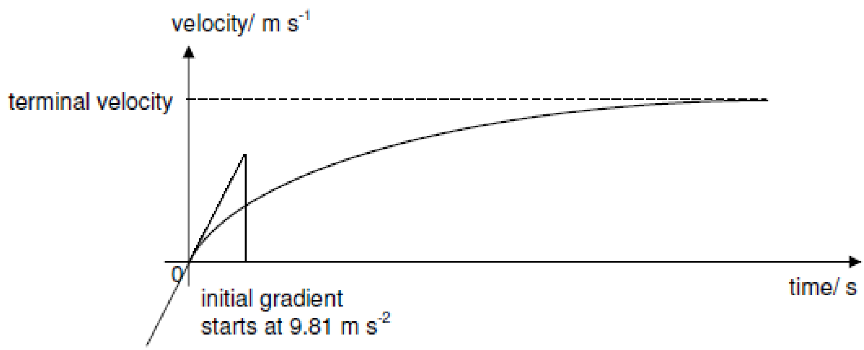
$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Note that for a particle in free-fall without air resistance, $a = -g = -9.81$ m s^{-2}.

Moreover, if air resistance is not negligible, then a particle in free-fall will experience a resistive force proportional to its velocity. As it falls down, the magnitude of the resultant force will get smaller and smaller, until eventually a constant velocity is attained. This constant velocity is called the **terminal velocity**.

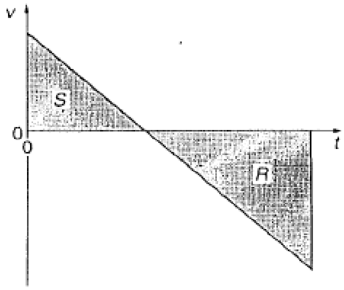


Example 2.1: The velocity of a car which is decelerating uniformly changes from 30 m s^{-1} to 15 m s^{-1} in 75 m. After what further distance will it come to rest?

Solution: By using $v^2 = u^2 + 2as$, we have $15^2 = 30^2 + 2 \times a \times 75$, and so $a = -4.5$ m s^{-2}.

Using the same formula again, $0^2 = 15^2 + 2 \times (-4.5) \times s$ will give $s = 25$ m.

Example 2.2: A stone is thrown upwards from the top of a cliff. After reaching its maximum height, it falls past the cliff-top and into the sea. The graph shows how the vertical velocity v of the stone varies with time t after being thrown upwards. R and S arethe areas of the two triangles.



What is the height of the cliff-top above the sea?

Solution: Note that the area under the $v - t$ graph from $t = 0$ to $t = t_0$ represents displacement of the stone from the initial point at time $t = t_0$. Here, S represents displacement at the highest point, and R represents displacement of the stone at the sea as measured from the highest point.

The height of the cliff-top above the sea is then $R - S$.

Example 2.3: A cricketer throws a ball vertically upward into the air with an initial upwards velocity of 18.0 m s^{-1}. How high does the ball go? How long is it before it turns to the cricketer's hands?

Solution: The displacement can be found by using the formula $v^2 = u^2 + 2as$, with $v = 0, a = -g$ and $u = 18.0$, which gives $s = 16.5$ m.
To find the total time, we use the formula $s = ut + \frac{1}{2}at^2$, with $u = 18.0, a = -9.8$ and $s = 0$. Here $s = 0$ because the ball returns to the same point at which it was thrown. Therefore, $t = 3.7$ s.

Exercise 2A

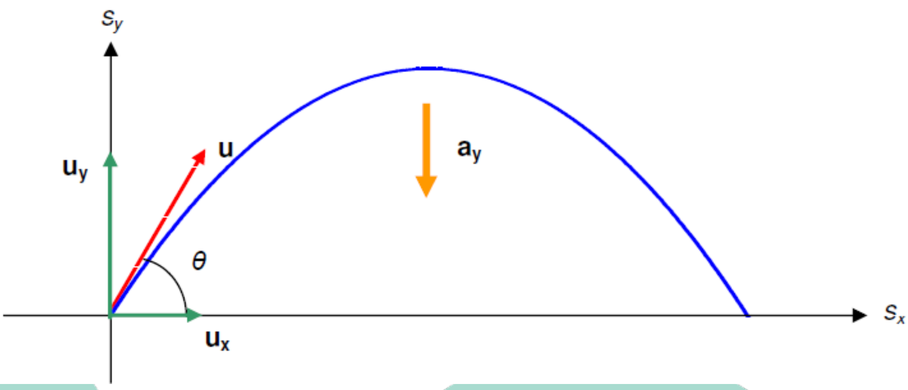
1. An aircraft travels 1600 km in 2.5 hours. What is its average speed in m s^{-1}?
   Ans: 180 m s^{-1}.

2. An airliner must reach a speed of 110 m s^{-1} to take off. If the available length of the runway is 2.4 km and the aircraft accelerates uniformly from rest at one end, what minimum acceleration must be available if it is to take off?
   Ans: 2.5 m s^{-2}.

3. A speeding motorist passes a traffic police officer on a stationary motorcycle. The police officer immediately gives chase: his uniform acceleration is 4.0 m s^{-2}, and by the time he draws level with the motorist he is travelling at 30 m s^{-1}.
   How long does it take for the police officer to catch the car? If the car continues to travel at a steady speed during the chase, what is that speed?
   Ans: 7.5 s, 15 m s^{-1}.

2. Projectile Motion

If a point particle is launched at an angle with respect to the horizontal, the trajectory of the particle will be a **parabola**.



To analyse such motion, we consider the horizontal and vertical component of the motion **separately**. Let u_x and u_y be the x and y-component of the initial velocity (which means $u_x = u\cos\theta$ and $u_y = u\sin\theta$), and x and y be the horizontal and vertical component of the displacement, respectively.
In case of a free-fall, there is an acceleration of $a_y = -g$ in the downward direction. Therefore,

$$\begin{cases} v_x = u\cos\theta \\ v_y = u\sin\theta - gt \end{cases}$$

and

$$\begin{cases} x = (u\cos\theta)t \\ y = (u\sin\theta)t - \frac{1}{2}gt^2 \end{cases}$$

Note that $v_y = 0$ at the **maximum height** of the projectile, and $y = 0$ when the projectile lands.

At any instant, the **magnitude of velocity** $v = \sqrt{v_x^2 + v_y^2}$ and $\tan\theta = v_y/v_x$.

By using the above equations, we can derive several general results. For instance, the **maximum height** is given by:

$$H = \frac{u^2\sin^2\theta}{2g}$$

The **time of flight** is:

$$t = \frac{2u\sin\theta}{g}$$

The **horizontal range** of the object is:

$$R = \frac{u^2\sin 2\theta}{g}$$

from which we deduce that the range is maximum when the projectile is launched at an angle of 45° with respect to the horizontal.

Example 2.4: A long-jumper leaves the ground at an angle of 20° to the horizontal and at a speed of 11 m s-1. How far does he jump? What is his maximum height reached?

Solution: First, we find the time when the jumper reached the ground again, i.e. $y = 0$:

$$0 = (u \sin 20°)t - \frac{1}{2}(9.81)t^2$$

which gives

$$t = \frac{2 \times 11 \sin 20°}{9.81} = 0.767 \text{ s.}$$

So, horizontal range is$x = (11 \cos 20°) \times 0.767 = 7.93$ m.
The maximum height is:

$$y = \frac{11^2 \sin^2 20°}{2 \times 9.81} = 0.721 \text{ m.}$$

Example 2.5: An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers. If the plane is travelling horizontally at 40.0 m s-1 and is 100 m above the ground, where does the package strike the ground relative to the point at which it was released?

Solution: First, we find the time when the package hits the ground, i.e. when $y = -100$ m. Note $u_y = 0$.

$$-100 = 0 - \frac{1}{2} \times 9.81 \times t^2$$

which gives $t = 4.52$ s. Therefore, $x = 40 \times 4.52 = 181$ m.

Example 2.6: An electron, travelling with a velocity of 2.0×10^7 m s-1 in a horizontal direction, enters a uniform electric field. This field gives the electron a constant acceleration of 5.0×10^{15} m s-2 in a direction perpendicular to its original velocity. The field extends for a horizontal distance of 60 mm. What is the magnitude and direction of the velocity of the electron when it leaves the field?

Solution:

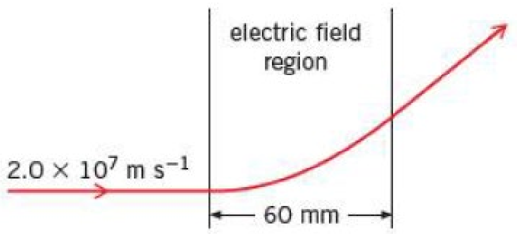


Figure 3.22

The horizontal motion of the electron is not accelerated. The time t spent in the field is given by $t = x/u_x = 3.0 \times 10^{-9}$ s. When the electron enters the field, $u_y = 0$. In time t, it has been accelerated to $v_y = u_y + at = 0 + 5.0 \times 10^{15} \times 3.0 \times 10^{-9} = 1.5 \times 10^7$ m s-1.
Also, $v_x = u_x = 2.0 \times 10^7$ m s-1. The resultant velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(2.0 \times 10^7)^2 + (1.5 \times 10^7)^2} = 2.5 \times 10^7 \text{ m s}^{-1}.$$

The direction this resultant velocity makes to the horizontal is θ, where $\tan\theta = v_y/v_x$, giving the answer $\theta = 37°$.

Exercise 2B

1. A ball is thrown horizontally from the top of a tower 30 m high and lands 15 m from its base. What is the ball's initial speed?
   Ans: 6.1 m s^{-1}.

2. A football is kicked on level ground at a velocity of 15 m s^{-1} at an angle of 30° to the horizontal. How far away is the first bounce?
   Ans: 20 m.

3. William was faced with the agonising task of shooting an apple placed on his son Jemmy's head. Assume that William is placed 25 m from Jemmy; his crossbow fires a bolt with an initial speed of 45 m s^{-1}. The crossbow and apple are on the same horizontal line. At what angle to the horizontal should William aim so that the bolt hits the apple?
   Ans: 3.5°.

END OF CHAPTER 2 EXERCISES

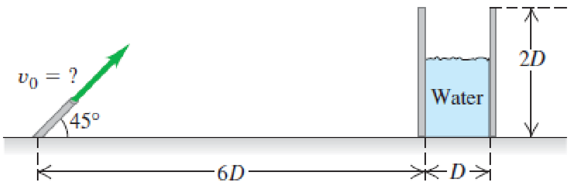
1. From the top of a tall building you throw one ball straight up with speed v_0, one ball straight down with speed v_0, one ball horizontally with speed v_0, and one ball at an angle of $45°$ with respect to the horizontal with speed v_0.
   a. Which ball has the greater speed when it reaches the ground?
   b. Which ball gets to the ground first?
   c. Which ball has the greatest displacement when it reaches the ground?

2. At the instant the traffic light turns green, a car that has been waiting at an intersection starts ahead with a constant acceleration of 3.20 m/s^2. At the same instant a truck starts to move with a constant speed of 20.0 m/s.
   a. Sketch the displacement-time and the velocity-time graph of the motion of both vehicles.
   b. How far beyond its starting point does the car overtake the truck?
   c. How fast is the car traveling when it overtakes the truck?

3. A hot-air balloonist, rising vertically with a constant velocity of magnitude 5.00 m/s, releases a sandbag at an instant when the balloon is 40.0 m above the ground.
   a. Compute the position and velocity of the sandbag at 1.00 s after its release.
   b. How many seconds after its release will the bag strike the ground?
   c. What is the greatest height above the ground that the sandbag reaches?

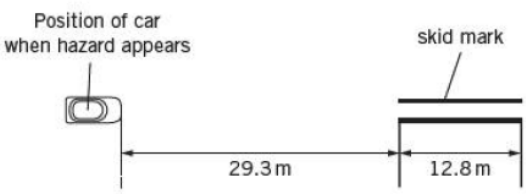
4. A test rocket is launched by accelerating it along a 200-m incline at 1.25 m/s^2 starting from rest. The incline rises at 35° above the horizontal, and at the instant the rocket leaves it, its engines turn off and it is subject only to gravity. Ignoring air resistance,
   a. Find the maximum height above the ground that the rocket reaches.
   b. Find the greatest horizontal range of the rocket, measured from the initial point.

5.



A water hose is used to fill a large cylindrical storage tank of diameter D and height $2D$. The hose shoots water at $45°$ above the horizontal from the same level as the base of the tank and is a distance $6D$ away. For what range of launch speeds (v_0) will the water enter the tank? Express your answer in terms of D and g.

6. **(O/N 2008 P2 Q2)** A car is travelling along a straight road at speed v. A hazard suddenly appears in front of the car. In the time interval between the hazard appearing and the brakes on the car coming into operation, the car moves forward a distance of 29.3 m. With the brakes applied, the front wheels of the car leave skid marks on the road that are 12.8 m long, as illustrated in the figure.



It is estimated that, during the skid, the magnitude of the deceleration of the car is $0.85g$, where g is the acceleration of free fall.

(a) Determine

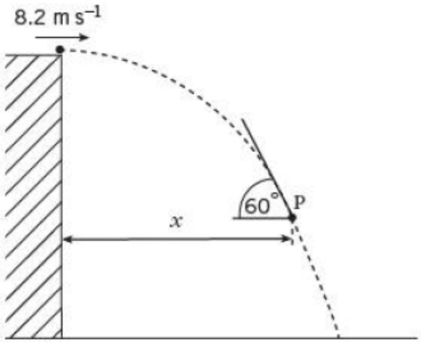
(i) the speed v of the car before the brakes are applied, [2]

(ii) the time interval between the hazard appearing and the brakes being applied. [2]

(b) The legal speed limit on the road is 60 km per hour.

Use both of your answers in (a) to comment on the standard of the driving of the car. [3]

7. **(O/N 2010 P21 Q2)** A ball is thrown horizontally from the top of a building, as shown below.



The ball is thrown with a horizontal speed of 8.2 m s^{-1}. The side of the building is vertical. At point P on the path of the ball, the ball is at a distance x from the building and is moving at an angle of 60° to the horizontal. Air resistance is negligible.

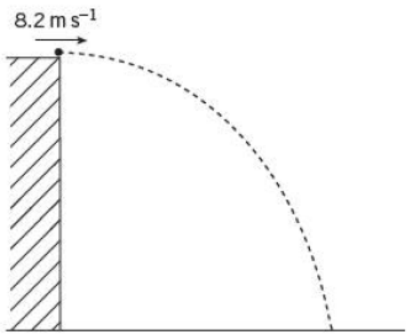
(a) For the ball at point P,

(i) show that the vertical component of its velocity is 14.2 m s^{-1}, [2]

(ii) determine the vertical distance through which the ball has fallen, [2]

(iii) determine the horizontal distance x. [2]

(b) The path of the ball in (a), with an initial horizontal speed of 8.2 m s^{-1}, is shown again below.

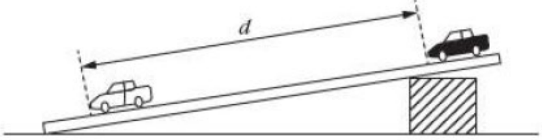


On the figure above, sketch the new path of the ball having an initial horizontal speed

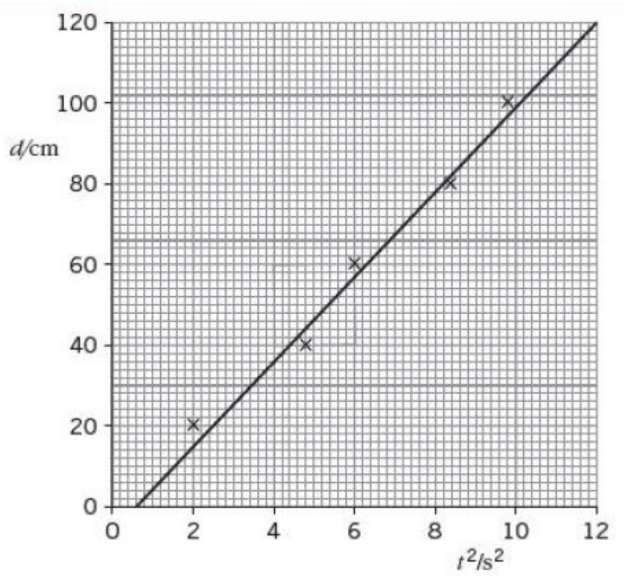
(i) greater than 8.2 m s^{-1} and with negligible air resistance (label this path G). [2]

(ii) equal to 8.2 m s^{-1} but with air resistance (label this path A). [2]

8. **(M/J 2004 P2 Q3)** A student has been asked to determine the linear acceleration of a toy car as it moves down a slope. He sets up the apparatus as shown in the figure below.



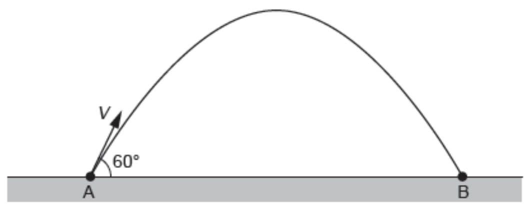
The time t to move from rest through a distance d is found for different values of d. A graph of d (y-axis) is plotted against t^2 (x-axis) as shown below.



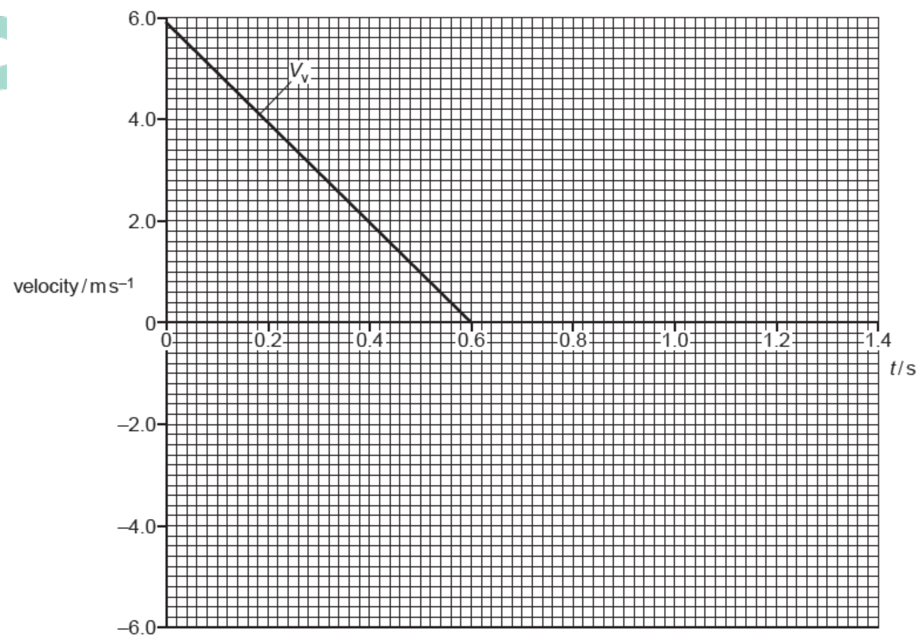
(a) Theory suggests that the graph is a straight line through the origin.
Name the feature on the above figure that indicates the presence of
(i) random error,
(ii) systematic error. [2]

(b) (i) Determine the gradient of the line of the graph above. [2]
(ii) Use your answer to (i) to calculate the acceleration of the toy. Explain your working. [3]

9. **(O/N 2014 P22 Q2)** A ball is thrown from A to B as shown below.



The ball is thrown with an initial velocity V at 60° to the horizontal.
The variation with time t of the vertical component V_v of the velocity of the ball from $t = 0$ to $t = 0.60$ s is shown in the diagram below.



Assume the air resistance is negligible.

(a) (i) Complete the figure above for the time until the ball reaches B. [2]

(ii) Calculate the maximum height reached by the ball. [2]

(iii) Calculate the horizontal component, V_h of the velocity of the ball a time $t = 0$. [2]

(iv) On the above figure, sketch the variation with t of V_h. Label this sketch V_h. [1]

(b) The ball has mass 0.65 kg. Calculate, for the ball,

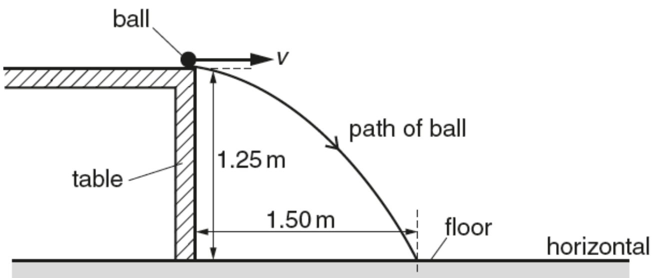
(i) the maximum kinetic energy, [3]

(ii) the maximum potential energy above the ground. [2]

10. **(M/J 2017 P22 Q2)**

(a) Define *velocity*. [1]

(b) A ball of mass 0.45 kg leaves the edge of a table with a horizontal velocity v, as shown below.



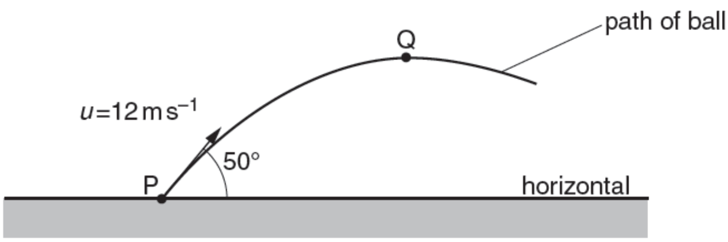
The height of the table is 1.25 m. The ball travels a distance of 1.50 m horizontally before hitting the floor. Air resistance is negligible. Calculate, for the ball,

(i) the horizontal velocity v as it leaves the space. [3]

(ii) the velocity (magnitude and angle) just as it hits the floor. [4]

11. **(M/J 2016 P21 Q2)**

A ball is thrown from a point P with an initial velocity u of 12 m s^{-1} at 50° to the horizontal, as illustrated below.



The ball reaches maximum height at Q. Air resistance is negligible.

(a) Calculate

(i) the horizontal component of u, [1]

(ii) the vertical component of u. [1]

(b) Show that the maximum height reached by the ball is 4.3 m. [2]

(c) Determine the magnitude of the displacement PQ. [4]